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PROPAGATION OF ONE-DIMENSIONAL ELASTOPLASTIC WAVES IN SOILS

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In the present article, proceeding from stress–strain theory [1], we investigate the distribution of a plane and a spherical wave in an elastoplastic medium. The stress–strain state of the medium is characterized by the displacement u(r, t), the strains $\varepsilon_{rr} = \partial u/\partial r$, $\varepsilon_{\varphi\varphi} = \varepsilon_{\theta\theta} = u/r$ ($\varepsilon_{\varphi\varphi} = \varepsilon_{\theta\theta} = 0$ in the planar case), and the stresses σ_{rr}, $\sigma_{\varphi\varphi} = \sigma_{\theta\theta}$. We show that either a shock wave or continuous loading–unloading waves can occur in the medium (soil), depending on the forms of the constitutive functions $\sigma(\varepsilon)$, $\sigma_i(\varepsilon_i)$ in the theory of [1]. The indicated waves in soils are investigated in the case $\sigma = (\alpha_1 + \alpha_2|\varepsilon|)\varepsilon$, $\sigma_i = (\beta_1 - \beta_2\varepsilon_i)\varepsilon_i$, where α_i, β_i (i = 1, 2) are positive constant coefficients. The solutions of the problems are obtained by an inverse approach [2, 3] with the geometry of the wave surface specified by a second-degree polynomial with respect to the time t (for a shock wave) or the coordinate r (for an unloading wave). It is assumed that the unloading process of the medium is irreversible and linear both with respect to the hydrostatic pressure σ with respect to the stress intensity σ_i. The parameters of the medium, including the load profile, are calculated on a computer on the basis of the derived analytical equations, and the results are presented as graphs of the components of the stresses and particle velocity. We also analyze the case $\sigma_i = \sigma_i(\varepsilon, \varepsilon_i)$ with regard for possible wave effects and the mutual influence of the first and second invariants of the stress or strain tensor. This study represents a continuation of [4] to the case where the strength characteristics of the medium are incorporated in the analysis of the dynamics of transient processes.

We note that problems in the propagation of a plane and a spherical wave have been studied previously by many authors, specifically in [5-14]. However, the soil and rock models used in those works differ considerably from [1]. For example, stress–strain theory is used in [5, 14], the constitutive equations of plastic flow [16] are used in [6-8], the theory of soil plasticity [17] is used in [9, 10], etc.

In contrast with [5-14], for our solution of the above-indicated problems we describe the motion and state of the medium under dynamic loading by the equations of the stress–strain theory of soil plasticity [1], demonstrate the existence of a plane unloading wave for a triaxial stressed state of the medium, and give detailed comparisons of the parameters of an elastoplastic medium and a generalized "plastic gas." We investigate the characteristic features of the propagation of a spherical wave in an elastoplastic medium and the behavior of its parameters for strong disturbances of an explosive nature.

1. Let an instantaneously initiated and then arbitrarily decaying load $\sigma_0(t)$ act along the normal to some plane. In this situation the equation of motion of the medium and the relations between the stresses and strains [1] with regard for the unloading theorem of Il'yushin [15] have the form

$$\rho_0\partial^2u/\partial t^2 = \partial\sigma_{rr}/\partial r; \tag{1.1}$$

in loading

$$\begin{gathered}\sigma_{rr} = (\lambda + 2G)\varepsilon,\ \sigma_{\varphi\varphi} = \sigma_{\theta\theta} = \lambda\varepsilon,\ \lambda = \sigma/\varepsilon - (2/9)\sigma_i/\varepsilon_i,\\ G = (1/3)\sigma_i/\varepsilon_i,\\ \sigma = (\alpha_1 + \alpha_2|\varepsilon|)\varepsilon,\ \sigma_i = (\beta_1 - \beta_2\varepsilon_i)\varepsilon_i;\end{gathered} \tag{1.2}$$

in unloading

$$\begin{gathered}\sigma_{rr} - \sigma^*_{rr} = (\lambda_0 + 2G_0)(\varepsilon - \varepsilon^*),\\ \sigma_{\varphi\varphi} - \sigma^{**}_{\varphi\varphi} = \lambda_0(\varepsilon - \varepsilon^*),\ \lambda_0 = E_1 - \frac{2}{9}E_2,\quad G_0 = \frac{1}{3}E_2,\end{gathered} \tag{1.3}$$

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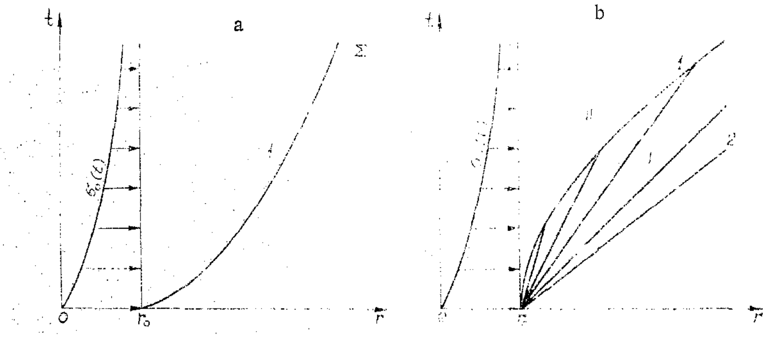

Fig. 1

where ε, ε_i, σ, σ_i are the first and second invariants of the strain and stress tensors; E_1 and E_2 are the slopes of the unloading branches [tangents of the angles of inclination of the respective curves $\sigma(\varepsilon)$ and $\sigma_i(\varepsilon_i)$ and the ε and ε_i axes]; and ε^*, σ^*_{rr}, $\sigma^*_{\varphi\varphi}$ are the strain and components of the stress at the start of unloading. Substituting (1.2) into (1.1) and making use of the relation $\varepsilon_i = -(2/3)\varepsilon$ $(\varepsilon > 0)$, we have

$$\rho_0 \frac{\partial^2 u}{\partial t^2} = \left[\left(\alpha_1 + \frac{4}{9}\beta_1\right) - 2\left(\alpha_2 - \frac{8}{27}\beta_2\right)\varepsilon\right] \frac{\partial^2 u}{\partial r^2}. \tag{1.4}$$

It is evident from (1.4) that for $(\alpha_2 - (8/27)\beta_2) > 0$ a shock wave $r = R^*(t)$ propagates in the soil (Fig. 1a, curve 1) with a velocity $a_1 = \{[\alpha_1 + (4/9)\beta_1 - 2(\alpha_2 - (8/27)\beta_2)\varepsilon]/\rho_0\}^{1/2}$ exceeding the longitudinal elastic wave velocity; otherwise, for $(\alpha_2 - (8/27)\beta_2) < 0$ centered loading waves propagate in the medium, where they are intercepted above by the unloading wave $t = f^*(r)$ (Fig. 1b, curve 1), which forms the boundary of demarcation between the regions of loading and unloading of the medium.

The case $\alpha_2 - (8/27)\beta_2 = 0$ leads to the "degenerate" elastic problem, whose solution is elementary.

We consider the case in which $(\alpha_2 - (8/27)\beta_2) > 0$. It is assumed here that the medium unloads after the shock front. The conditions at the shock front and at the boundary of the loading plane have the form

$$\sigma_{rr} = -\rho_0 \dot{R}^*(t)\dot{u}^*, \quad \dot{u}^* = -\dot{R}^*(t)\varepsilon_{rr},$$
$$\dot{R}^*(t) = dR^*(t)/dt, \quad \dot{u}^* = \partial u^*/\partial t \ \text{for} \ r = R^*(t); \tag{1.5}$$
$$\sigma_{rr} = -\sigma_0(t) \ \text{for} \ r = r_0, \ t \geqslant 0. \tag{1.6}$$

Inasmuch as the solution of the problem is formulated inversely, it is assumed that the velocity $\dot{R}^*(t)$ of propagation of the front (shock waveform) is given, and in the course of solving the problem expression (1.6) is used to determine the load profile $\sigma_0(t)$.

Let $\dot{R}$ be given; then at $r = R^*(t)$ expression (1.5) has the form

$$\varepsilon^*(t) = -\frac{\rho_0 \dot{R}^{*2}(t) - \left(\alpha_1 + \frac{4}{9}\beta_1\right)}{\left(\alpha_2 - \frac{8}{27}\beta_2\right)}, \quad \frac{\partial u^*}{\partial t} = -\dot{R}^*(t)\,\varepsilon^*(t). \tag{1.7}$$

Now, substituting (1.3) into (1.1), we obtain the equation

$$\frac{\partial^2 u}{\partial t^2} = a_0^2 \frac{\partial^2 u}{\partial r^2} - a_0^2 \frac{\partial \varepsilon^*(r)}{\partial r} + \frac{1}{\rho_0}\frac{\partial \sigma^*_{rr}(r)}{\partial r},$$

where $\rho_0 a_0^2 = \lambda_0 + 2G_0$; this equation admits the solution

$$u(r,t) = f_1(r - a_0 t) + f_2(r + a_0 t) - \frac{1}{\rho_0 a_0^2}\int_{r_0}^{r}\left[\sigma^*_{rr}(r) - \rho_0 a_0^2 \varepsilon^*(r)\right] dr. \tag{1.8}$$

We note that if the equation $r = R^*(t)$ holds and $\dot{R}^*(t)$ is represented in relation to r, then the strain ε^* and the particle velocity $\partial u^*/\partial t$ in (1.7) will be functions of the coordinate r. With the application of (1.7) the unknown functions f_1 and f_2 have the form

$$f_i'(z_i) = \frac{(-1)^i}{2a_0}\left\{\frac{\dot{R}^*(F_i(z_i))\left[\rho_0 \dot{R}^{*2}(F_i(z_i)) - \left(\alpha_1 + \frac{4}{9}\beta_1\right)\right]}{\left(\alpha_2 - \frac{8}{27}\beta_2\right)} + (-1)^i \frac{\sigma^*_{rr}[R^*(F_i(z_i))]}{\rho_0 a_0}\right\}, \tag{1.9}$$

where $F_i(z_i)$ $(i=1, 2)$ is the root of the equation $R^*(t) \pm a_0 t = z_i$ in relation to t and the prime denotes the derivative with respect to the argument.

From (1.1), making use of (1.6), (1.8), and (1.9), we obtain an equation for determining the load profile:

$$\sigma_0(t) = -\sigma_{rr}^*[R^*(t)] + \rho_0 \int_{r_0}^{R^*(t)} \frac{\partial^2 u}{\partial t^2} dr. \tag{1.10}$$

On the basis of (1.8) and (1.10) we have carried out calculations for a specific form of $R^*(t)$.

For $[\alpha_2 - (8/27)\beta_2] < 0$ the perturbed zone in the (r, t) plane is partitioned into regions of loading (I) and unloading (II) of the medium (see Fig. 1b), and they are demarcated by the unloading wave surface $t = f^*(r)$. In the region of active deformation in the soil after the shock front $r = at$, where $a = \sqrt{(\alpha_1 + (4/9)\beta_1)/\rho_0}$ (curve 2), centered Riemann waves propagate, which are closed by a region of constant parameters near the boundary of the loading plane. In the given situation the problem is solved for a stepped load. Since the curves $\sigma(\varepsilon)$ and $\sigma_i(\varepsilon_i)$ do not have elastic intervals, the displacement and strain of the medium have zero values rather than discontinuities at the front $r = at$.

To formulate a solution of this problem we introduce the self-similar variable $\xi = r/t$, whereupon we obtain from (1.4)

$$\rho_0 \xi \frac{du_t}{d\xi} + \left[\left(\alpha_1 + \frac{4}{9}\beta_1\right) - 2\left(\alpha_2 - \frac{8}{27}\beta_2\right)\varepsilon\right]\frac{d\varepsilon}{d\xi} = 0,$$
$$du_t/d\xi + \xi d\varepsilon/d\xi = 0 \quad (u_t = \partial u/\partial t). \tag{1.11}$$

Next, setting the determinant of the homogeneous system (1.11) equal to zero, we have

$$\xi = \sqrt{\frac{(\alpha_1 + (4/9)\beta_1) - 2(\alpha_2 - (8/27)\beta_2)\varepsilon}{\rho_0}}. \tag{1.12}$$

Now, with regard for (1.12), the particular solutions of the system (1.11) acquire the form

$$\varepsilon = -\frac{\rho_0(\xi^2 - \xi_0^2)}{2\left(\alpha_2 - \frac{8}{27}\beta_2\right)}, \quad u_t = \frac{\rho_0(\xi^3 - \xi_0^3)}{3\left(\alpha_2 - \frac{8}{27}\beta_2\right)}, \tag{1.13}$$

where $\rho_0\xi_0^2 = (\alpha_1 + (4/9)\beta_1)$. The condition $\sigma_{rr} = -\sigma_0 = \text{const}$ at $\xi = \xi_k$ enables us to determine the boundary of the region in which centered waves propagate in the soil; its equation is written in the form

$$\xi_k^2 = \xi_0^2 \sqrt{1 + \frac{\sigma_0\left(\alpha_2 - \frac{8}{27}\beta_2\right)}{(\rho_0\xi_0^2/2)^2}}.$$

The problem in the unloading region II (Fig. 1b) is reducible to the determination of the functions f_1 and f_2 of expression (1.8) and the unloading waveform $t = f^*(r)$ with satisfaction of (1.6) and the condition

$$\partial u/\partial r = \varepsilon^*(r), \quad \partial u/\partial t = \partial u^*/\partial t \quad \text{for} \quad t = f^*(r), \tag{1.14}$$

where $\varepsilon^*(r)$, $\partial u^*/\partial t$ are known functions given by (1.13).

We note that (1.6) can be replaced by the strain condition, namely that for $r = r_0$, $t \geq 0$;

$$\partial u/\partial r = \varepsilon_0(t). \tag{1.15}$$

Then, substituting (1.8) into (1.14) and (1.15) and carrying out suitable transformations, we obtain a system of equations in f_2' and $f^*(r)$ of the form

$$f_2'[r + a_0 f^*(r)] - f_2'[(2r_0 - r) + a_0 f^*(r)] + \frac{\sigma_{rr}^*(r_0) - \sigma_{rr}^*(r)}{\rho_0 a_0^2} - \varepsilon^*(r_0) + \varepsilon_0\left[\frac{(r_0 - r) + a_0 f^*(r)}{a_0}\right] = 0,$$
$$a_0\left\{f_2'[r + a_0 f^*(r)] + f_2'[(2r_0 - r) + a_0 f^*(r)] - \frac{\sigma_{rr}^*(r_0)}{\rho_0 a_0^2} + \varepsilon^*(r_0) - \varepsilon_0\left[\frac{(r_0 - r) + a_0 f^*(r)}{a_0}\right]\right\} = \frac{\partial u^*}{\partial t}. \tag{1.16}$$

From (1.16), eliminating f_2', we obtain

$$\frac{\sigma_{rr}^*(r_0)}{\rho_0 a_0^2} - \varepsilon^*(r_0) + \varepsilon_0\left(\frac{z - r_0}{a_0}\right) - \frac{1}{2}\sum_{i=1}^{2}\left[\frac{\sigma_{rr}^*(F_i(z))}{\rho_0 a_0^2} + \frac{(-1)^i}{a_0}\frac{\partial u^*}{\partial t}(F_i(z))\right] = 0, \tag{1.17}$$

where $F_i(z)$ $(i=1, 2)$ denotes the roots of the respective equations $(2r_0-r)+a_0f^*(r)=z$ and $r+a_0f^*(r)=z$ in r. Making use of the fact that the velocity and waveform of the unloading wave depend on the loading strain $\varepsilon^*(r)$, i.e., $r/f^*(r)=a_U(\varepsilon^*)$, we infer from (1.17) that if the distribution of remanent strains $\bar{\varepsilon}(r)$ is known from experiment, the relation $\bar{\varepsilon}(r)=\varepsilon^*(r)-\sigma^*(\varepsilon^*)/E_1$ is used to calculate $\varepsilon^*(r)$ and then determine the unloading waveform, so that the strain distribution and then the stress distribution in the loading plane can be determined by means of (1.17).

Let us assume that the unloading wave form $t=f^*(r)$ is given, and let it be required to determine $\sigma_0(t)$ in the course of solving the problem in region II of the (r, t) plane. Now (1.14) takes the role of the boundary conditions for finding the unknown functions f_1 and f_2 given in (1.8). Denoting by $F_i(\xi_i)$ the roots of the equation $r\mp a_0f^*(r)=\xi_i$ $(i=1, 2)$ in r, from (1.14) we find

$$f_i'(\xi_i)=\frac{1}{2}\left\{\frac{\sigma_{rr}^*[F_i(\xi_i)]}{\rho_0a_0^2}\mp\frac{1}{a_0}\frac{\partial u^*}{\partial t}[F_i(\xi_i)]\right\}.$$

Then for the determination of $\varepsilon_0(t)$ we have

$$\varepsilon_0(t)=\frac{1}{2}\sum_{i=1}^{2}\left\{\frac{\sigma_{rr}^*[F_i(r_0+(-1)^ia_0t)]}{\rho_0a_0^2}+\frac{(-1)^i}{a_0}\frac{\partial u^*}{\partial t}[F_i(r_0+(-1)^ia_0t)]\right\}-\frac{\sigma_{rr}^*(r_0)}{\rho_0a_0^2}+\varepsilon^*(r_0).$$

From the strain $\varepsilon_0(t)$, taking (1.3) into account, we find the stress $\sigma_{rr}(r_0, t)$ and then the load profile $\sigma_0(t)=-\sigma_{rr}(r_0, t)$.

2. In the case of a spherical wave, for $(\alpha_2-(8/27)\beta_2)>0$ the solution of the equation of motion of the medium

$$\frac{\partial^2u}{\partial t^2}=a_0^2\left[\frac{\partial^2u}{\partial r^2}+\frac{2}{r}\left(\frac{\partial u}{\partial r}-\frac{u}{r}\right)+\frac{Q(r)}{\rho_0a_0^2}\right],\ \text{where}$$

$$Q(r)=\frac{\partial}{\partial r}\left[\sigma_{rr}^*(r)-\rho_0a_0^2\varepsilon^*(r)\right]+\frac{2}{r}\left[\sigma_{rr}^*(r)-\sigma_{\varphi\varphi}^*(r)-2G_0\varepsilon^*(r)\right], \tag{2.1}$$

subject to (1.7), has the following form when $R^*(t)$ is given as an arbitrarily decaying time function:

$$\begin{aligned}u(r,t)=\frac{1}{r}\Bigg\{&\int_{r_0}^{r-a_0t}d\xi_2\int_{r_0}^{\xi_2}\Phi(\xi_1)\,d\xi_1-\int_{r_0}^{r+a_0t}d\xi_2\int_{r_0}^{R^*(F_2(\xi_2))-a_0F_2(\xi_2)}\Phi(\xi_1)d\xi_1+\\&+\int_{r_0}^{r+a_0t}\frac{R^*(F_2(\xi_2))}{(\lambda_0+2G_0)}d\xi_2\int_{r_0}^{R^*(F_2(\xi_2))}Q(\xi_1)\,d\xi_1-\int_{r_0}^{r+a_0t}\frac{R^*(F_2(\xi_2))\left[\rho_0\dot{R}^{*2}(F_2(\xi_2))-\left(\alpha_1+\frac{4}{9}\beta_1\right)\right]}{\left(\alpha_2-\frac{8}{27}\beta_2\right)}d\xi_2\Bigg\}-\\&-\frac{1}{r^2}\Bigg\{\int_{r_0}^{r-a_0t}d\xi_3\int_{r_0}^{\xi_3}d\xi_2\int_{r_0}^{\xi_2}\Phi(\xi_1)\,d\xi_1-\frac{a_0r_0^2\left(1+\frac{\dot{R}^*(0)}{a_0}\right)\left[\rho_0\dot{R}^{*2}(0)-\left(\alpha_1+\frac{4}{9}\beta_1\right)\right]}{\left(\alpha_2-\frac{8}{27}\beta_2\right)}t-\\&-\int_{r_0}^{r+a_0t}d\xi_3\int_{r_0}^{\xi_3}d\xi_2\int_{r_0}^{R^*(F_2(\xi_2))-a_0F_2(\xi_2)}\Phi(\xi_1)\,d\xi_1-\int_{r_0}^{r+a_0t}d\xi_3\int_{r_0}^{\xi_3}\frac{R^*(F_2(\xi_2))\left[\rho_0\dot{R}^{*2}(F_2(\xi_2))-\left(\alpha_1+\frac{4}{9}\beta_1\right)\right]}{\left(\alpha_2-\frac{8}{27}\beta_2\right)}d\xi_2+\\&+\int_{r_0}^{r+a_0t}d\xi_3\int_{r_0}^{\xi_3}\frac{R^*(F_2(\xi_2))}{(\lambda_0+2G_0)}d\xi_2\int_{r_0}^{R^*(F_2(\xi_2))}Q(\xi_1)\,d\xi_1\Bigg\}-\frac{r}{3(\lambda_0+2G_0)}\int_{r_0}^{r}Q(r)\,d\xi+\frac{1}{3(\lambda_0+2G_0)r^2}\int_{r_0}^{r}\xi^3Q(\xi)\,d\xi,\end{aligned} \tag{2.2}$$

where

$$\Phi(z_1)=\frac{1}{2[a_0-\dot{R}^*(F_1(z_1))]\left(\alpha_2-\frac{8}{27}\beta_2\right)}\Bigg\{\Bigg[\dot{R}^*(F_1(z_1))\Bigg(1+$$

$$+2\frac{\dot{R}^*(F_1(z_1))}{a_0}\Bigg)-a_0+R^*(F_1(z_1))\frac{\ddot{R}(F_1(z_1))}{a_0}\Bigg]\left[\rho_0\dot{R}^{*2}(F_1(z_1))-\left(\alpha_1+\frac{4}{9}\beta_1\right)\right]+2\rho_0R^*(F_1(z_1))\dot{R}^*(F_1(z_1))\ddot{R}^*(F_1(z_1))\times$$

$$\times\left(1+\frac{\dot{R}^*(F_1(z_1))}{a_0}\right)\Bigg\}+\frac{1}{2(\lambda_0+2G_0)}\int\limits_{r_0}^{R^*(F_1(z_1))}Q(\xi)\,d\xi-\frac{R^*(F_1(z_1))\,\dot{R}^*(F_1(z_1))\,Q[R^*(F_1(z_1))]}{2[a_0-\dot{R}^*(F_1(z_1))](\lambda_0+2G_0)};$$

the functions $F_i(z_i)$ $(i=1, 2)$ are the roots of the equations $R^*(t)\pm a_0 t = z_i$ in t.

Differentiating (2.2) with respect to t and r, we determine the particle velocity $\dot{u}(r, t)$ and the strain $\varepsilon(r, t)$, and then on the basis of (1.3) we find the stress components σ_{rr}, $\sigma_{\varphi\varphi}$. In determining Q(r), however, as in the case of a plane wave, we must express the strain ε^* at the shock front as a function of r. To do so we need to solve the equation $r=R^*(t)$ for t and substitute the result into (1.7). We then obtain $\varepsilon^*(r)$ and, making use of (1.2) and (2.1), find $\sigma^*_{rr}(r)$, $\sigma^*_{\varphi\varphi}(r)$ and Q(r).

In the case $(\alpha_2-(8/27)\beta_2)<0$ the behavior of the spherical problem in the loading region I (Fig. 1b) differs from the plane case in that the problem is not self-similar, the centered waves in the soil turn out to be curvilinear, and the parameters of the medium along their surface are variables. Analytical procedures are therefore unsuitable for obtaining simple solutions of the problem. Accordingly, to solve the spherical problem in the loading zone, as in [2, 14], we use the method of characteristics. Below, we analyze the plastic stress–strain state of the medium after the unloading wave front (Fig. 1b) in region II in the same way as in Sec. 1. We shall not write out the cumbersome analytical solutions.

3. Experimental studies [18-22] of the mechanical properties of soils under triaxial compression conditions at an elevated stress level evince the validity of the deformation equation $\sigma_i=\sigma_i(\varepsilon, \varepsilon_i)$, in the majority of situations, whereas the relationship between σ and ε is autonomous. In unloading, the interactions of the invariants σ, σ_i, ε, ε_i are slight, and so $\sigma=\sigma(\varepsilon, \varepsilon^*, \sigma^*)$, $\sigma_i=\sigma_i(\varepsilon_i, \varepsilon_i^*, \sigma_i^*)$, where the starred parameters correspond to the start of unloading. An investigation of the nature of waves in soils with suitable approximations of $\sigma_i=(\varepsilon, \varepsilon_i)$, in particular for $\sigma_i=3\sigma(\gamma_1\varepsilon_i-\gamma_2\varepsilon_i^2)$, shows that a shock wave occurs in the zone of active loading of the soil.

In this case, solving the inverse problem for determining the deformation $\varepsilon^*(t)$ on a shcok-wave front, we obtain a transcendental equation – in particular, a 3rd-degree polynomial, which is solved numerically using a standard procedure. The solutions in the discharge region are the same as in Secs. 1 and 2, so we do not repeat them here.

4. We have carried out calculations for the cases in which the geometry of the surfaces of the shock wave and unloading wave are specified in the form of second-degree polynomials:

$$R^*(t)=r_0+R_1t-(R_2/2)t^2,\ \dot{R}^*(t)>0; \tag{4.1}$$

$$f^*(r)=B_0(r-r_0)-(B_1/2)(r-r_0)^2,\ \dot{f}^*(r)>0 \tag{4.2}$$

and the initial parameters of the medium have the values

$$\rho_0=0.02\cdot10^4\ \mathrm{kg\cdot sec^2/m^4},\ \sigma_0(0)=105\cdot10^4\ \mathrm{kg/m^2},\ r_0=0.1\ \mathrm{m}, \tag{4.3}$$

$$R_1=420\ \mathrm{m/sec}\ R_2=2\cdot10^2R_1,\ B_0=0.2927\cdot10^{-2}\ \mathrm{sec/m}\ B_1=2\cdot10^{-1}B_0;$$

$$\alpha_1=12.127\cdot10^6\ \mathrm{kg/m^2},\ \alpha_2=58.73\cdot10^7\ \mathrm{kg/m^2}, \tag{4.4}$$

$$\beta_1=35.83\cdot10^6\ \mathrm{kg/m^2},\ \beta_2=11.64\cdot10^8\ \mathrm{kg/m^2},\ E_1=14\cdot10^7\ \mathrm{kg/m^2},$$

$$E_2=2\cdot10^7\ \mathrm{kg/m^2}.$$

The results of computer calculations are shown in Figs. 2-5 in the form of curves of the stresses, particle velocity, and load as a function of the time in the cross sections $r=0.1$, 0.2 and at the shock front $r=R^*(t)$. The solid curves in Figs. 2-4 refer to the case of a plane shock wave propagating in an elastoplastic soil for (4.3) and (4.4); the dashed curves with crosses refer to the case $\beta_2=-11.64\cdot10^8$ kg/m^2, corresponding to the "shock" curve $\sigma_i=\sigma_i(\varepsilon_i)$; and the plain dashed curves correspond to a generalized plastic gas [4].

In the case of the generation of a plane unloading wave in soil, i.e., for $\beta_2=28.75\cdot10^7$ kg/m^2, the variation of the load profile $\sigma_0(t)$ is shown in Fig. 5. It is evident from Fig. 2 that in the inverse problem the profile of the unknown load $\sigma_0(t)$ on the chamber $r=0.1$ in the case of a generalized plastic gas varies slowly with t in comparison with the theory of elastoplastic deformations. This is because the medium is compressed from all sides with uniform pressure when the problem is analyzed on the basis of the generalized plastic gas model, whereas with application of the dynamical theory of plasticity $\sigma_{rr}>\sigma_{\varphi\varphi}=\sigma_{\theta\theta}$, and so the process of decay of $\sigma_0(t)$ is comparatively rapid in the latter case. The curve $\sigma_0(t)$ for $d\sigma_i/d\varepsilon_i>0$, $d^2\sigma_i/d\varepsilon_i^2>0$ passes between the curves $\sigma_0(t)$ calculated for $d^2\sigma_i/d\varepsilon_i^2<0$ and on the basis of an ideal medium and is situated close to the curve $\sigma_0(t)$ for the generalized plastic. The curves of the stress $\sigma_{\varphi\varphi}$, being monotonically decreasing time functions, become tensile for $t\geq0.086\cdot10^{-3}$ sec $(d^2\sigma_i/d\varepsilon_i^2<0)$ and $t\geq0.164\cdot10^{-3}$ $(d^2\sigma_i/d\varepsilon_i^2>0)$.

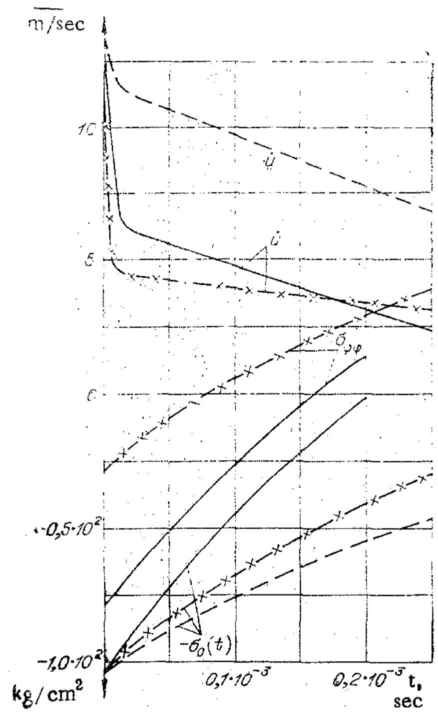

Fig. 2

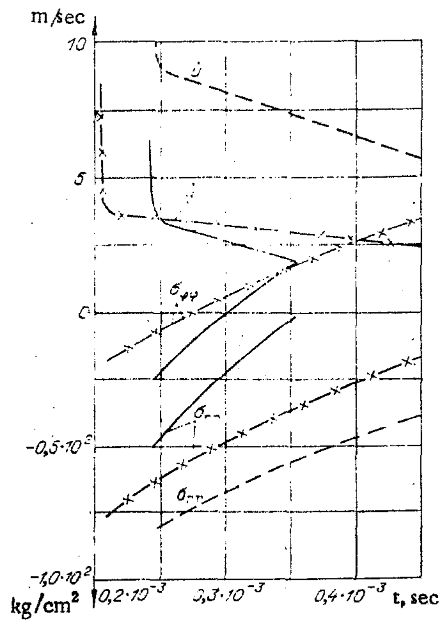

Fig. 3

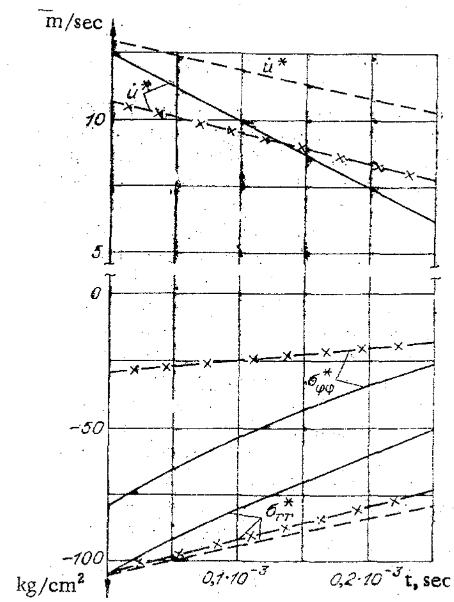

Fig. 4

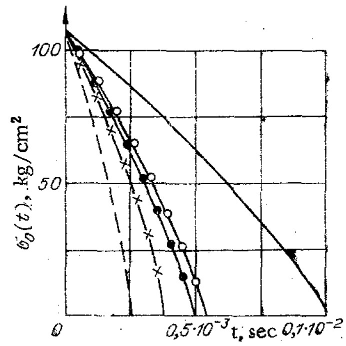

Fig. 5

The behavior of the particle velocity $\dot{u}$ as a function of t turns out to be very nearly similar in all of the cases investigated above and has a decaying trend. The overall decrease of $\dot{u}$ in the vicinity of the shock front in the region of unloading of the medium subjected to elastoplastic deformations is greater than in the plastic gas. However, the value of $\dot{u}$ is a maximum for the plastic gas case.

The qualitative pattern of the variation of the parameters σ_{rr}, $\sigma_{\varphi\varphi}$, $\dot{u}$ at $r = 0.2$ (Fig. 3) is similar to the case $r = 0.1$ with allowance for their quantitative values and the arrival time of the wave at a given point.

Unlike the region of unloading of the medium at the shock front $r = R^*(t)$, the stresses $\overset{*}{\sigma}_{rr}$, $\overset{*}{\sigma}_{\varphi\varphi}$ and the particle velocity $\dot{u}^*$ vary roughly linearly with t. This is because the wave velocity $\dot{R}^*(t)$ is specified in the calculations as a linearly decreasing function of the time.

The investigation has shown that in order to obtain the same load on the chamber in elastoplastic and ideal media it is necessary that the nature of the variation (decrease) of $\dot{R}^*(t)$ in the elastoplastic medium be mild and protracted, and so the stress field will have a prolonged destructive force and an elongated wave pattern in comparison with the ideal medium. When the "shock diagrams" of the medium are used, despite the fact that the profiles $\sigma_0(t)$ for the elastoplastic and ideal media do not differ appreciably, their velocities in the unloading region and along the shock front will differ considerably.

Figure 5 shows the load profile $\sigma_0(t)$ obtained from the solution of the problem when the plane unloading wave given by Eq. (4.2) is generated in soil. Here the solid, dashed, and dot-dash curves correspond to relations between the coefficients of (4.2) $B_2 = 2 \cdot 10^{-1} B_0$, $B_1 = 8 \cdot 10^{-1} B_0$, and $B_1 = 4 \cdot 10^{-1} B_0$. Also, the dashed curves with crosses and circles are plotted for the cases $E_1 = 7 \cdot 10^7$ kg/m^2 and $E_2 = 16 \cdot 10^7$ kg/m^2 ($B_1 = 4 \cdot 10^{-1} B_0$). It is evident from these curves that the curvature of the load profile is directly proportional to that of the unloading wave (4.2), i.e., the curve $\sigma_0(t)$ is steeper for a larger ratio (stronger coupling) between B_1 and B_0. With a reduction in the Young's modulus E_1 the load and the time of its action on the chamber are decreased accordingly, whereas in the case of shock propagation [4] the reverse is true. The same pattern is observed when the Young's modulus E_2 is varied. However, the effect of E_2 on $\sigma_0(t)$ is not as pronounced as that of E_1.

We note that similar studies involving an analysis of the structure and classification of one-dimensional (plane and spherical) waves in solids can be carried out for any approximation of the constitutive functions $\sigma(\varepsilon)$, $\sigma_i(\varepsilon_i)$, $\sigma_i(\varepsilon, \varepsilon_i)$ in the stress–strain theory of plasticity.

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